Cutthroat Trout Length Conversion Regressions

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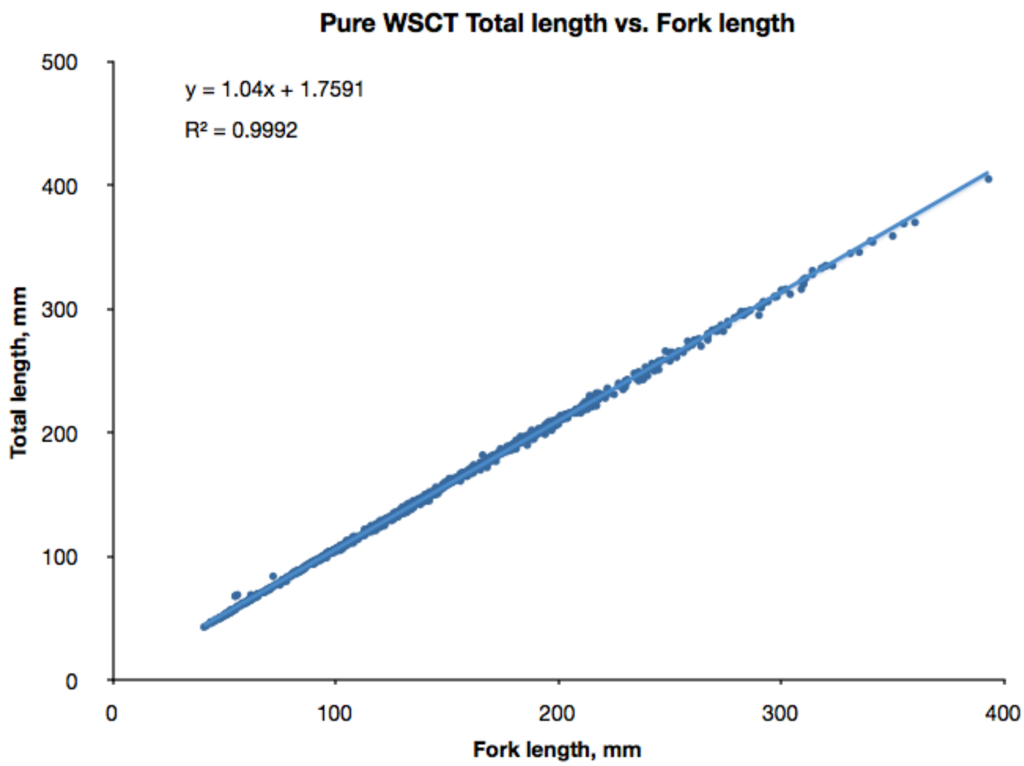

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The problems posed for biologists by the real world seldom, if ever, have an exact, correct statistical solution. The assumptions of nearly all techniques are violated to a greater or lesser extent by real data. That does not, and should not, stop scientists from using the best available methods to extract useful, if sometimes slightly unreliable, information. In the case of regression for description (not prediction) in the presence of unmeasured or unmeasurable variation on all the variables involved, researchers should be aware that no method gives a correct, unbiased result with real data.

McArdle 1988:2338

Minor corrections and additions, October 2012: included discussion of Chen et al. (1994) and Chen and Jackson (2000).

Abstract

Reliable methods are needed to translate different measurements of trout length into a common form for use in further analyses. I used paired fork length and total length data extracted from the > 59,000 cutthroat trout entries in the Alberta Fish and Wildlife Management Information System (FWMIS) database to compute ordinary least squares (OLS) and functional geometric mean (GM) linear regression equations for converting between total length and fork length for (a) all Alberta fish identified as cutthroat trout regardless of their subspecies (westslope or Yellowstone) or genetic status (hybrid or pure), and (b) westslope cutthroat trout for which there is reliable evidence that the fish are genetically pure. No statistically significant or practical differences were found between OLS and GM equations for the various conversions, but the GM regressions (below) are recommended because they should be somewhat better for predictions beyond the length ranges from which they were derived.

cutthroat trout in general:

FL = 0.961 • TL - 2.061, N = 14,278, range 25 - 506 mm TL, individual prediction SE ± 1.88 mm (1)

TL = 1.040 • FL + 2.144, N = 14,278, range 24 - 491 mm FL, individual prediction SE ± 1.95 mm (2)

genetically-pure Alberta westslope cutthroat trout:

FL = 0.961 • TL - 1.631, N = 926, range 43 - 405 mm TL, individual prediction SE ± 1.84 - 1.85 mm (3)

TL = 1.040 • FL + 1.697, N = 926, range 41 - 393 mm FL, individual prediction SE ± 1.91 - 1.93 mm (4)

where FL is fork length, TL is total length, N is the sample number, and the standard error is for predicted FL or TL *for individual fish* given TL or FL, respectively. Confidence intervals for predictions can be obtained by multiplying the prediction standard errors by $t_{\alpha,N-2}$.

Each pair of equations is actually the same equation, each one an algebraic rearrangement of the other. The regressions for cutthroats in general (equations 1 and 2), and for genetically-pure westslope cutthroats (equations 3 and 4), have identical slopes for comparable conversions, while their intercepts differ significantly ($p < 0.05$), but by just 0.43 mm and 0.45 mm for conversions to FL and TL, respectively. Whether these differences reflect a real morphological difference between pure westslope cutthroats and non-native or hybridized stocks, or differential measurement errors, cannot yet be determined.

Although differences in predicted lengths are small (tenths of mm) and negligible for most purposes, users of these regressions should employ equations 3 and 4 and their associated prediction standard errors on known pure westslope cutthroats; otherwise equations 1 and 2 and their associated prediction standard errors should be employed. Standard errors for those wishing to estimate other types of confidence limits are provided (such as for the predicted mean Y of a given X), and are subject to the same recommendation.

Analysis of Studentized residuals showed that outliers were at least 5 times more frequent than expected. These may have been due to one or more of recording and transcription errors, differences in measurement methods in certain collections, or real morphological differences in certain populations. The regression analyses reported here can be used to develop controls for automated error trapping during data entry in the FWMIS forms. This approach could significantly improve data quality in the existing database of cutthroat trout already in the system if used to screen the existing file, as well as improve the accuracy of newly-entered data in the future. Despite the large number of outliers, low Cook's D values demonstrated that these regressions were not significantly influenced by them.

Introduction

Fishery biologists often must relate one measure to another as a basic part of their day-to-day work. Length-to-weight conversions and various conversions among different measures of length are common examples. These conversions are nearly always done by computing ordinary least-squares (OLS) linear regressions (with or without transformation of variables) with the aid of standard statistical packages, or with built-in least squares regression functions in commercial spreadsheets.

Packaged OLS linear regression calculations are based on a particular model that is designed to be used on data with quite specific characteristics. In particular, it is applicable to data in which there is one dependent measurement for each independent measurement, and in which the independent variable is either assigned, or at least is measured with negligible error. It has long been recognized that many biological applications of regression analysis do not fit the assumptions of this model, and that other models are more appropriate in such cases (Teissier 1948, Simpson *et al.* 1960). This is certainly the case with many applications in fisheries (Ricker 1973).

Ricker (1973) examined the issue at considerable length, and concluded that, in common fisheries situations where both variates are measured with error or are inherently variable, have non-normal and/or open-ended distributions, a functional regression rather than the much more frequently used OLS regression is more suitable, the latter producing biased estimates to some degree. He suggested that using the geometric mean (GM) functional regression was more appropriate in most such situations, *even when the object is prediction*. He provided calculation methods for the GM regression parameters (from Teissier 1948), their variances, and methods to calculate approximate confidence limits, as well as a useful tabular key (Ricker 1973:Table 8) for determining what regression models are most appropriate for different situations. In a response to a critique (Jolicoeur 1975), he later gave previously published expressions for determining exact asymmetric confidence limits of the slope of a GM regression (Ricker 1975a). Still later, he published updated information on the GM regression method, providing a detailed explication of its advantages in many situations, including corrections to the 1973 paper (Ricker 1984). He drew particular attention to its lack of dependence on scale, and its robustness: it can be used, with caution, for extrapolation.

McArdle (1988) considered OLS, GM and other approaches to linear estimation when both variables were measured with error, and concluded that though both OLS and GM approaches were reliable and complementary overall, when OLS is unreliable due to violations of assumptions, GM and related methods are appropriate substitutes.

Chen et al. (1994) evaluated the performance of several regression techniques in a simulation study, and reanalyzed the data of several OLS-based fisheries studies using GM and so-called “robust” regression techniques [least absolute value (LAV), least median of squares (LMS), and the LMS-based reweighted least squares (RLS)] with particular attention to performance when outliers were present in one or both variables — a common situation in fisheries work.

They found GM and LAV to be somewhat more accurate than OLS in such cases, but that all three provided much poorer estimates of slope and intercept than did LMS or RLS. They recommended a two-step analysis, using LMS to identify outliers for study as to their cause and possible correction or elimination, and using RLS to estimate parameters from the corrected or cleaned-up dataset. Later, two of the same authors (Chen and Jackson 2000) showed in another simulation and fisheries data reanalysis that selecting the most suitable regression procedure depends very much on the detailed error structure of the data. In cases where outliers caused by both substantial variance as well as bias are present in both variables, GM often provides the most accurate estimates. In cases where the error structure cannot be determined, they proposed that their LMS-based reweighted geometric mean (RGM) approach be used.

More recently the GM approach as been extended to multivariate analyses (Tofallis 2003), and others have advanced new methods to deal with the problem of errors-in-variables (Gillard and Iles 2009). These generally require that the amount of variable error be known (usually it is not in fisheries problems).

Use of the GM regression is recommended by some standard texts (Ricker 1975b, Anderson and Neumann 1996) for calculating length conversions; however the latter authors inexplicably do not mention it for calculating weight-length regressions even though the arguments in its favour for that purpose still hold (Ricker 1973, 1975b; but see Jensen 1986). Use of GM regression has been dismissed as an *ad hoc* approach in a recent treatment of methods in relative abundance estimation (Hubert and Fabrizio 2007) based on others' opinions (Hilborn and Walters 1992). The latter authors considered it in relation to stock-recruitment analyses, which are especially prone to large measurement errors, and which have serious consequences in that application. Hilborn and Walters (1992) felt Ricker's functional regression approach to resolving errors-in-variables problems (such as those in many simple linear regression analyses) was not widely applied because (1) it is only applicable for single-variable (*sic*) regressions, which excludes most common stock-assessment techniques, and (2) it is an *ad hoc* method with no well-understood ability to correct for bias. They noted that some highly complex calculations had been developed for correcting for the errors-in-variables bias (apparently in other, non-GM models; Ludwig and Walters 1981), but that they "do not work particularly well" (Hilborn and Walters 1992:234). Hubert and Fabrizio (2007:306) proposed that jackknife, bootstrap[1], simulation and randomization[2] approaches may be used to estimate precision of regression parameters. Elsewhere the same authors note (Hubert and Fabrizio 2007:298) that jackknife and bootstrap techniques used for this purpose "will not usually yield the same answer", and similar to Chen and Jackson's findings above, "there is no agreement on which technique is 'better'." No examples or references are offered for the simulation approach, and the sole example

[1] LMS is a bootstrap-related method; see discussion of Chen et al. above.

[2] Those authors support the randomization suggestion with a reference to Casey and Myers (1998), the correct bibliographic information for which is Canadian Journal of Fisheries and Aquatic Sciences 55:2329–2340.

suggested that uses the randomization approach did so only because the data presented unique problems (large diurnal variation in trawl catches) that precluded more usual methods.

A brief current review is available on the whole issue of linear estimation when there are errors in variables (Gillard 2010). Regrettably, it does not deal with the work of Chen et al (1994) and Chen and Jackson (2000) discussed above, and there seems to be little agreement on workable solutions as yet.

These considerations became an issue for me because I was recently asked to provide a means to convert between measurements of total length and fork length in cutthroat trout (J. Earle, personal communication). As an interim response, I provided a packaged ordinary least squares solution using a spreadsheet application. For a final solution I wanted to find the most appropriate model not only for this relatively simple problem, but for several other more difficult linear regression problems I intend to address in an upcoming report. While research scientists, statisticians and fisheries modelers sort out their objections to various simple linear regression models, I must fall back on the most workable existing advice. That appears to be provided by Ricker (1973, 1975a, 1975b, 1984), and Anderson and Neumann (1996) in the most current textbook on basic fisheries methods, as well as my reading of the results of Chen and Jackson (2000) as they pertain to the likely error structure in fork length vs. total length data of cutthroat trout. Specifically, both variables are likely to be subject simple measurement variance as well as occasional recording or transcription errors in one or the other.

Accordingly, this report compares GM functional linear regression and OLS linear regression solutions for converting between total length and fork length in Alberta cutthroat trout. Alberta cutthroats include the native westslope subspecies, introduced Yellowstone cutthroat subspecies, or various hybrids of these two taxa and with rainbow trout. For this reason I also compared GM functional and OLS linear regressions relating total length and fork length between known pure native stocks of westslope cutthroats and all cutthroat trout regardless of taxon or hybrid status. The objectives of the work were to provide the best available linear regression for converting between fork length and total length of Alberta trout commonly identified as cutthroat trout; to do the same for known genetically pure native westslope cutthroat trout; to compare the various regression results between regression models, and between all cutthroats and genetically-pure native cutthroats to clarify the consequences, if any, of using the different models on the two cutthroat groups.

Methods

I used the data on over 59,000 individual fish identified as cutthroat trout in the Fish and Wildlife Management Information System (FWMIS) database as of 16 December 2011. From this I extracted over 14,000 specimen entries for which both total length (TL) and fork length (FL) had been recorded. These were used to develop linear regressions of FL on TL and TL on FL for application to fish identified as cutthroat trout without regard to their genetic status or taxonomic identity (all CTTR). From that reduced database I then extracted the records for more than 900 trout from cutthroat populations identified as genetically pure, presumed native westslope cutthroat trout (pure WSCT). For these I again calculated simple linear regressions to convert between fork length and total length.

Specimens from genetically pure westslope cutthroat trout populations were identified as follows. I plotted the locations of trout collections of different genetic status on 1:50,000-scale National Topographic Service topographic maps in three categories (Table 1). Genetic data used for this classification came from extensive molecular genetic surveys (Taylor and Gow 2007, 2008; Janowicz 2010; Rasmussen *et al*. 2010; Mayhood and Taylor 2011; additional E. B. Taylor data to March 2011). These surveys are the best and most recent available for the study area. I then compared each of the capture locations of the > 14,000 cutthroat trout to the geographically plotted genetic data. Fish from, or close to, genetic sampling locations identified as pure were taken to be pure if the genetic sample size for that location equalled or exceeded 25 and there was no likelihood of contamination from known hybrid stocks in nearby connected waters.

Table 1. Criteria for classifying genetic purity on map plots. To be considered pure, minimum sample N ≥ 25 with no likelihood of contamination from nearby connected populations of lesser purity. For 1 exception, see text.

Purity criterion for mapping, this study	Fish & Wildlife SARCEP Hybridization Metric (Coombs, pers. comm.)	
Pure	≥ 0.99 mean purity	no hybrids, no risk of hybridizing fish entering population
	≥ 0.99 mean purity	no hybrids known, but proximity to hybridizing fish causes concern
Nearly Pure	≥ 0.99 mean purity	hybrids rare but have been detected or are strongly suspected
Introgressed	≥ 0.95 mean purity	some hybrids in most samples
	≤ 0.95 mean purity	mostly hybrids

So, for example, the upper Carbondale River trout were ruled out as possibly hybridized because only one of the upper stream's sampling locations had only pure fish and for that sample N = 16, there also being a potential contaminating hybrid source population upstream in MacDonald Creek, and others downstream but still above a barrier. On the other hand an exception, Syncline Brook, was considered pure even though only 11 fish were sampled because that may be almost the entire adult population, which is isolated above a barrier — a streambed that remains dry over a long distance except for a few days during particularly high freshets.

Linear regressions were calculated as described by Ricker (1973) and Sokal and Rohlf (1969). For convenience and clarity, the formulas and expressions are repeated here.

Ordinary Least Squares Regressions

X and Y are N pairs of the two types of length measurement linearly related by the equation

$$Y = bX + a \qquad (1)$$

where b is the slope, or regression coefficient, and a is the Y intercept.

$\bar{X}$ and $\bar{Y}$ are the arithmetic means of X and Y, respectively;

x and y are the differences of X and Y from their respective means (i.e., $x = X - \bar{X}$; $y = Y - \bar{Y}$); and s_x and s_y are the standard deviations of X and Y, respectively. The linear (Pearson) correlation coefficient between X and Y was determined as

$$r = \sum xy \,/\, (\sum x^2 \sum y^2)^{0.5} \qquad (2)$$

The ordinary least squares regression coefficient was calculated as

$$b = \sum xy \,/\, \sum x^2 \qquad (3)$$

and the intercept was found from either of the equations

$$a = \bar{Y} - b\bar{X} = (\sum Y - b\sum X)/N \qquad (4)$$

The variance of all points X_i, Y_i vertically from line (1) was taken as any of

$$s^2_{yx} = (\sum y^2 - (\sum xy)^2 / \sum x^2) \,/\, (N-2) = (\sum y^2 - b\sum xy) \,/\, (N - 2) = [\sum y^2 (1 - r^2)] \,/\, (N - 2) \qquad (5)$$

and the variance of the regression coefficient b is either of

$$s^2_{\boldsymbol{b}} = s^2_{yx} \,/\, \sum x^2 = [v^2 (1 - r^2)] \,/\, (N - 2), \qquad (6)$$

$$\text{where } v^2 = \sum y^2 / \sum x^2 \qquad (7)$$

The variance of a was calculated as[3]

$$s^2_{\boldsymbol{a}} = s^2_{yx} \,[(1/N) + (\bar{X}^2 / \sum x^2)] \qquad (8)$$

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[3] This is expression (8) in Ricker (1973:411). It is clear from his explanation that he means $\bar{X}^2$ and not X^2 as printed.

The variance of predicted $\hat{Y}_i$ for any given X_i was calculated differently depending on whether the intention was to predict the mean $\hat{Y}_i$ at length X_i, or the $\hat{Y}_i$ for a particular individual fish X_i. For a ***mean*** $\hat{\mathbf{Y}}_i$, the variance is (Ricker 1984:1899)

$$s^2_{yx} \left((1/N) + [(X_i - \bar{X})^2 / \sum x^2]\right) = s^2_{yx}/N + s^2_b (X_i - \bar{X})^2 \qquad (9)$$

For a ***single particular*** $\hat{\mathbf{Y}}_i$ estimated from a single X_i, the variance is (Ricker 1984:1899)

$$s^2_{yx} \left((1 + (1/N) + [(X_i - \bar{X})^2 / \sum x^2]\right) \qquad (10)$$

In all of the above cases, the standard error for each parameter estimate was calculated as the square root of the variance, allowing confidence limits to be assigned to the parameters in the usual way by multiplying the standard error by $t_{\alpha,N-2}$, where α is the confidence probability desired, and the degrees of freedom are N - 2.

All of the above calculations were done for the OLS regression of Y on X. The OLS regression of X on Y was also calculated to provide a means of obtaining length X from length Y as

$$X = dY + c \qquad (11)$$

$$\text{where } d = \sum xy / \sum y^2 = r^2/b \qquad (12)$$

All other statistics for (11) were calculated from equations (4) to (10) by interchanging x and y, X and Y in the applicable equations. Analyses of variance of ordinary least squares regressions were conducted in a spreadsheet (Numbers 2.1, Apple, Inc.) as described by Sokal and Rohlf (1969:421-423), and with SYSTAT 5.2.1 (Wilkinson 1992). SYSTAT was also used to compute residuals and related statistics for the ordinary least squares regression lines.

Geometric Mean Functional Regressions

Similar to ordinary least squares linear regressions, GM functional linear regression of Y on X on the same data takes the form

$$Y = vX + u \qquad (13)$$

where v and u are the slope and intercept, respectively. The GM regression coefficient v is calculated as any of

$$v = \pm (\sum y^2 / \sum x^2)^{0.5} = \pm (b/d)^{0.5} = \pm\ s_y/s_x = \pm\ b/r = \pm [b^2 + s^2_b (N - 2)]^{0.5} \qquad (14)$$

The intercept u was obtained as in (4) using v in place of b.

The GM regression coefficient of X on Y is $1/v$. Because there is only one GM regression line (rather than the two in ordinary least squares regression), the regression of X on Y can be obtained by algebraic rearrangement of equation (13) to solve for X, or it can be determined by interchanging the relevant terms in (14).

Variance for v is the same as for b, and the variance of u was estimated in an analogous fashion to that of a, obtainable from (6) and (8), respectively. Similarly, variances for predicted mean $\hat{Y}_i$ and predicted individual $\hat{Y}_i$ were calculated as for the OLS regression equivalents using (9) and (10), respectively.[4] Confidence limits for the parameters v and u, and for the GM regression predicted mean $\hat{Y}_i$ and predicted individual $\hat{Y}_i$ however, are asymmetric, so cannot be calculated in the usual way. Confidence limits for v are (Ricker 1975a, b)

$$v[(B + 1)^{0.5} \pm B^{0.5}] \quad (15)$$

where $B = F_{\alpha,1,N-2}(1 - r^2)/(N - 2)$. Ricker (1984) used $t^2_{\alpha,N-2}$, rather than $F_{\alpha,1,N-2}$; the coefficients are interchangeable. The correct asymmetrical confidence limits for u, predicted mean $\hat{Y}_i$ and predicted individual $\hat{Y}_i$ apparently have not yet been worked out, so these were approximated by using the symmetrical equivalents obtained by multiplying the respective variances by $t_{\alpha,N-2}$.

All calculations were done in a Numbers 2.1 spreadsheet and were checked against output from SYSTAT 5.2.1 where applicable. The spreadsheet model in Numbers and a translated version for Excel (Microsoft, Inc.) accompanies the electronic version of this document, and is also available by request from the author.

[4] Ricker (1973:414) presented a version of an expression attributed to Teissier (1948:32) for obtaining the variance of an individual predicted $\hat{Y}_i$, but deleted it without explanation 11 years later (Ricker 1984:1904). Ricker (1984:1902) suggested using (9) and (10) as minimum estimators of variance for mean predicted $\hat{Y}_i$ and individual predicted $\hat{Y}_i$, respectively, as I have done here. Expression (9) with X set to 0 provides (8), the approximation to the variance for the Y intercept u.

Results

The sample of all Alberta cutthroat trout ranged from 24 mm to 491 mm fork length (25 mm to 506 mm total length), the distribution being skewed toward larger sizes, with a long tail comprised of a relatively small number of large fish (Figures 1A, 2A). The sample of pure westslope cutthroat trout averaged almost 1 cm smaller with a narrower range — from 41 mm to 383 mm fork length (43 mm to 405 mm total length), but again showed an extended distribution tail of larger fish (Figures 1B, 2B). The variability of length distributions for the two groups were nearly identical: coefficients of variation 42.8% and 43.1% for all CTTR and pure WSCT, respectively. Small fish represented a declining percentage of the sample below about 120 mm fork length for all CTTR, and below about 200 mm fork length for pure WSCT, although there was a great deal of variation over the latter distribution (Figure 1B).

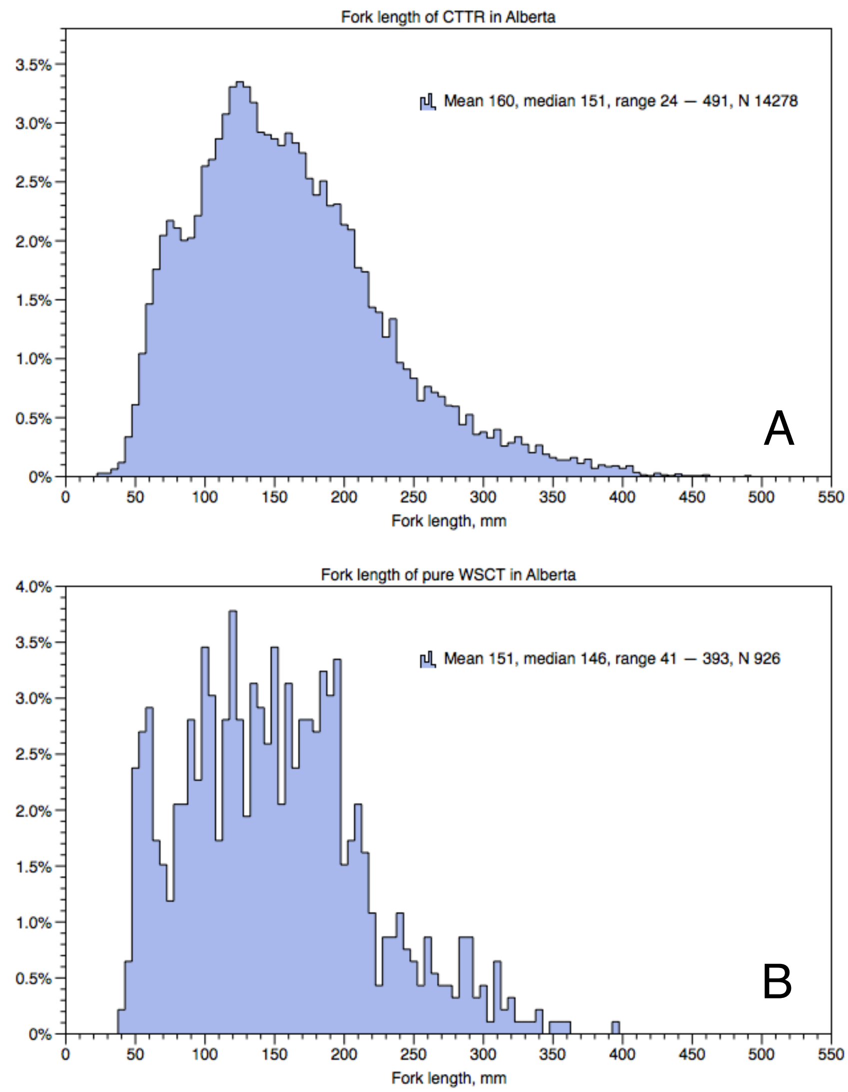

Figure 1. Size distribution by fork length for (A) all cutthroat trout, and (B) pure westslope cutthroat trout, Alberta.

Figure 2 illustrates the OLS regressions of total length on fork length for all CTTR and pure WSCT. The OLS regressions for fork length on total length, and the equivalent GM regressions, are all virtually identical to these, so are not illustrated. A number of outliers are apparent. These are much more obvious in the plots of Studentized residuals (Figure 3). Many exceed ± 3, especially in Figure 3A. That is, they lie more than ± 3 standard deviations from the best-fit line.

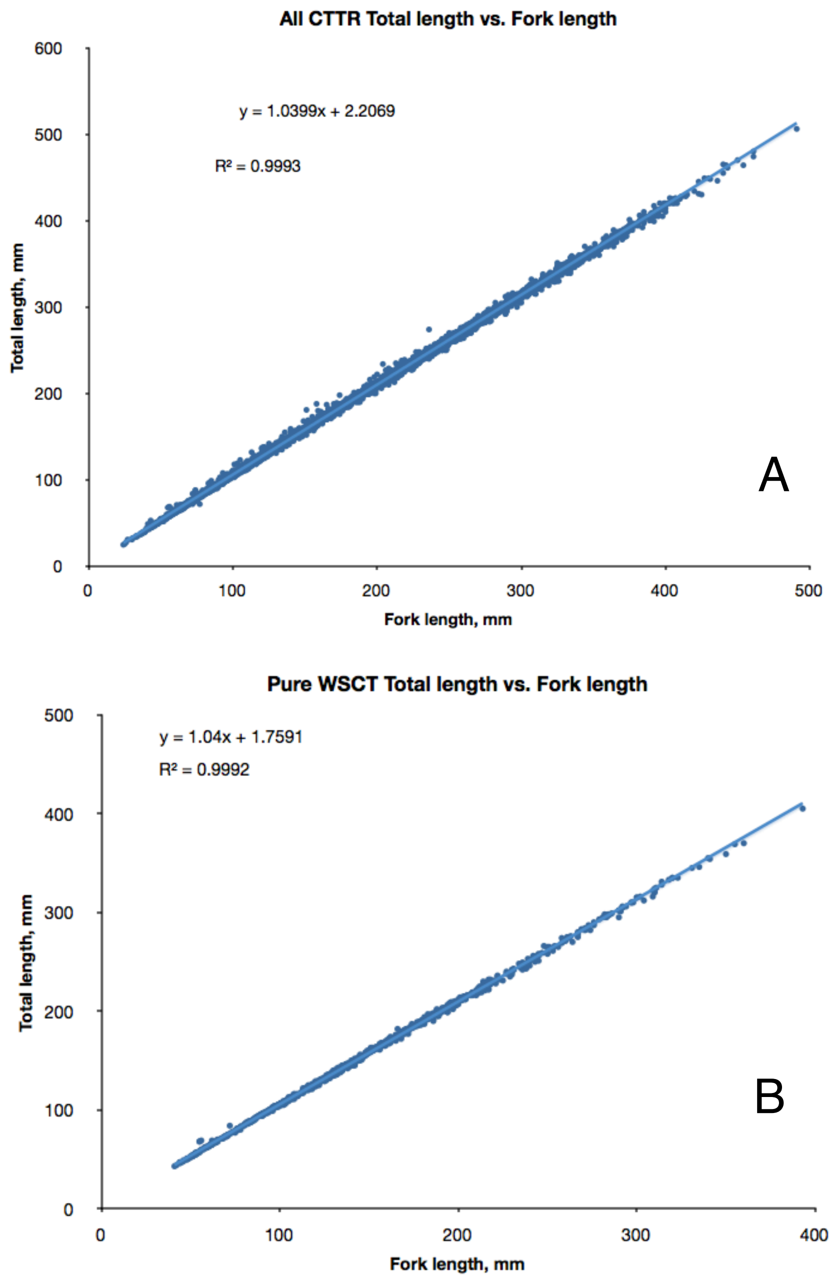

Figure 2. OLS regression of total length on fork length for (A) all cutthroat trout, and (B) pure westslope cutthroat trout, Alberta. Fork length on total length and the comparable GM regressions are very similar, so are not shown.

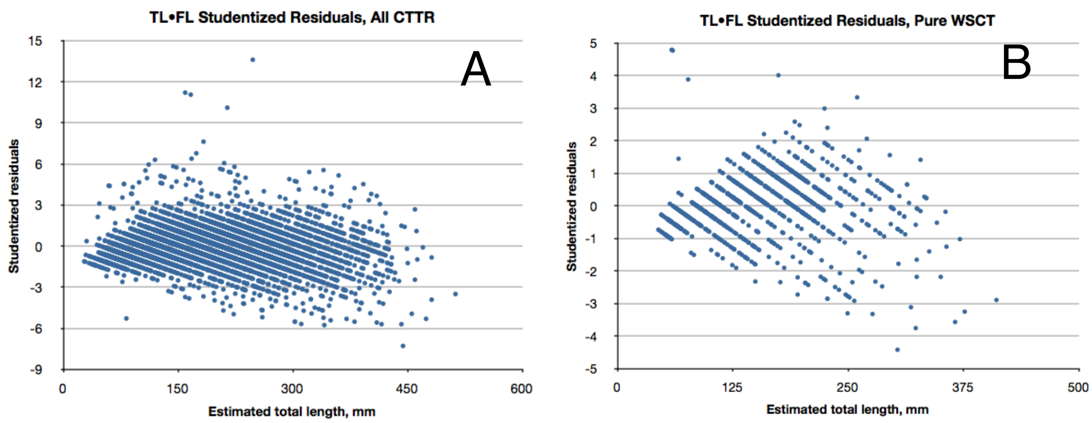

Figure 3. Studentized residuals for the total length on fork length OLS regressions (A) all cutthroat trout, and (B) pure westslope cutthroat trout, Alberta. Specimens with Studentized residuals ≥ ± 3 are rare; those ≥ ± 2.6 should be checked for possible errors in transcription or measurement. Residual plots of fork length on total length, and for the comparable GM regressions, are very similar, so are not shown.

Overall in the SYSTAT analysis, 317 potentially erroneous outliers (≥ 2.6 standard deviations from the line; probability of occurrence < 0.01) were flagged in the regressions for all CTTR (Appendix Table A1); 23 outliers were flagged in the regressions for pure WSCT (Appendix Table A2). Although 287 points in the dataset for all CTTR, and 11 points in the dataset for pure WSCT were flagged as having high leverage, the absolute values were low (<< 0.2) in all cases (Appendix Tables A1, A2). Cook's D considers leverage and the Studentized residual values together as a measure of a point's influence on the regression statistics, and is distributed as $F_{2,N\text{-}2}$. Cook's D was ≤ 0.03 (p = 0.97) in the regression for all CTTR (Figure 4A), and ≤ 0.071 (p = 0.93) in the regression for pure WSCT (Figure 4B), so did not trigger a flag in either SYSTAT analysis (Appendix Tables A1, A2).

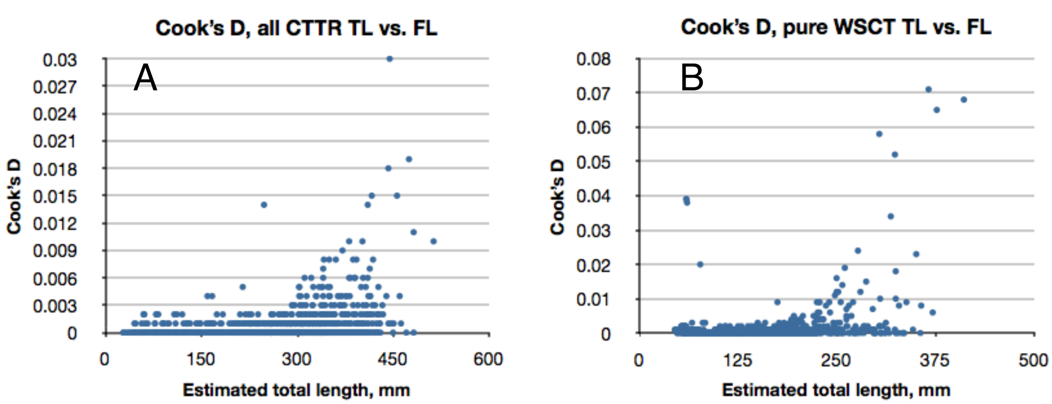

Figure 4. Cook's D for the total length on fork length OLS regressions (A) all cutthroat trout, and (B) pure westslope cutthroat trout, Alberta

One clearly erroneous outlier was discarded before final analysis from the dataset for all cutthroat trout. That datapoint was for a fish with a fork depth of about 7 cm for a trout less than 35 cm total length, an impossibly deep fork in the tail of a fish of that size. This was probably a recording or transcription error. Other outliers could not be so easily explained from the data at hand, and were retained, even though many are highly questionable.

Table 2 presents the OLS and GM regressions for converting between fork length and total length of cutthroat trout. Confidence limits for predicted lengths can be calculated as described in the caption for that table. For example, a single cutthroat of unknown genetic purity or subspecies having a fork length of 200 mm would be estimated using the GM regression

$$TL = 1.040 \bullet FL + 2.144$$

to have a total length of 210.1 mm. Multiplying the relevant standard error of prediction (SEP) of 1.95 by $t_{0.05,14276} = 1.960$ and applying the result to the estimate, the true value can be expected to lie within ± 3.8 mm of 210.1 mm (i.e., within the interval 206.3 — 213.9 mm) 19 times out of 20 (95% confidence interval). If the estimate is to answer the question,

*What is the total length, **on average**, of a cutthroat trout of fork length 200 mm?*

Table 2. Ordinary least squares regressions (OLS) and geometric mean functional regressions (GM) for predicting total length (TL) from fork length (FL) and FL from TL, all cutthroat trout and pure westslope cutthroat trout, Alberta. Standard errors of prediction for individual fish (single SEP) and for fish of a given length on average (mean SEP) are also provided. Confidence limits can be calculated for predicted lengths by multiplying the relevant SEP by $t_{\alpha,N-2}$. N = 14,278 for all CTTR, N = 926 for pure WSCT. Appendix Tables A3 to A5 provide SEPs by length class. The equations in the coloured cells are recommended (see Discussion).

Stock	Ordinary Least Squares Regression	Single SEP	Mean SEP	Geometric Mean Functional Regression	Single SEP	Mean SEP
all CTTR	TL = 1.040•FL + 2.207	1.95	0.02—0.08	TL = 1.040•FL + 2.144	1.95	0.02—0.08
all CTTR	FL = 0.961•TL - 2.001	1.88	0.02—0.08	FL = 0.961•TL - 2.061	1.88	0.02—0.08
pure WSCT	TL = 1.040•FL + 1.759	1.91—1.93	0.1—0.2	TL = 1.040•FL + 1.697	1.91—1.93	0.1—0.2
pure WSCT	FL = 0.961•TL - 1.570	1.84—1.85	0.1—0.2	FL = 0.961•TL - 1.631	1.84—1.85	0.1—0.2

then the applicable SEP is 0.02—0.08 (exactly 0.02; Appendix Table A3), and the total length estimate is 210.1 ± 0.04, or 210.06 — 210.14, 19 times out of 20. Tenths of a millimetre are rarely of any consequence in length measurements of trout, and hundredths of millimetres effectively are never of concern, so the SEPs for the estimates of mean values are of interest only to show that the errors associated with mean estimates are inconsequential, and that the mean estimate can be taken as perfectly precise.

The slopes do not differ among the several regressions (Figure 5); however there are differences among the intercepts (Figure 6). The intercepts for OLS and GM regressions are the same for comparable regressions, but they differ significantly between the regressions for all CTTR and pure WSCT. For any given fork length, the total length of pure WSCT is, on average, 0.45 mm less than that for cutthroat trout as a whole.

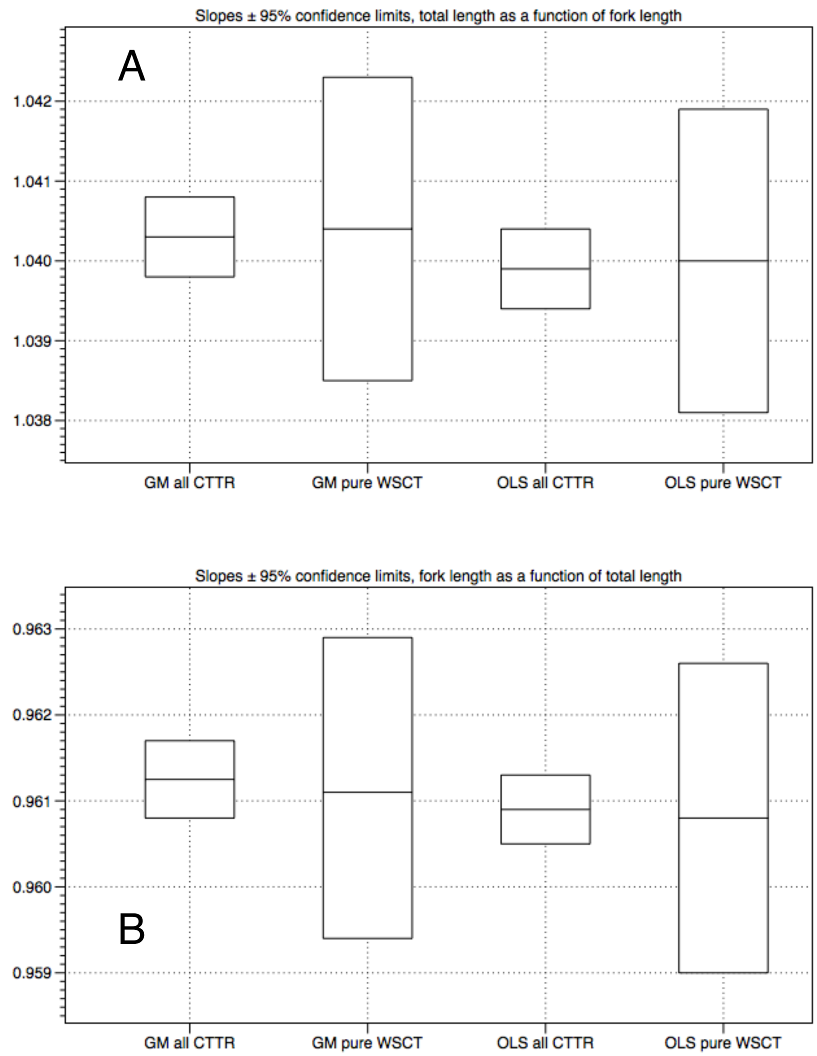

Figure 5. Slope *b* ± 95% confidence limits for the OLS and GM regressions all cutthroat trout, and pure westslope cutthroat trout, Alberta. A — total length on fork length; B — fork length on total length.

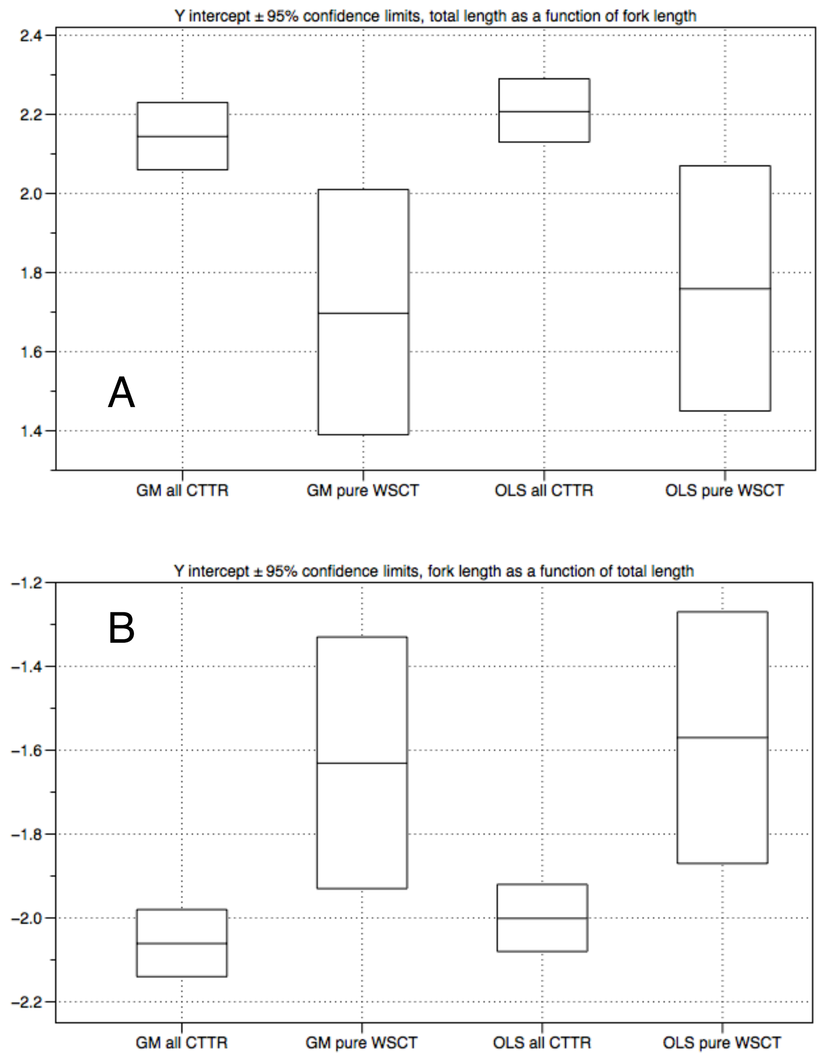

Figure 6. Intercept *a* ± 95% confidence limits for the OLS and GM regressions for all cutthroat trout, and pure westslope cutthroat trout, Alberta. A — total length on fork length; B — fork length on total length.

Discussion

Small fish are clearly under-represented in samples of both all CTTR and pure WSCT. A random sample representing self-sustaining populations must show much larger numbers of small fish than large fish to account for mortality, or even to explain the number of the medium- to large-sized fish found in the samples. This is likely to be much more of a problem for estimating the OLS regressions, which are quite sensitive to extreme values (Ricker 1973, 1984). The GM regressions, in contrast, are much less sensitive, even being robust enough to reliably predict beyond the range of the data used to develop the GM regressions.

The numerous Studentized residuals much greater than ± 3 point to numerous errors or unaccounted-for variations in the underlying data. For both pure WSCT and all CTTR, the probability of points occurring ≥ 3 standard deviations away from the estimated line is diminishingly small — 0.0027 for all CTTR; 0.0028 for pure WSCT. In the sample of 14,278 CTTR we should expect to find 38.6 specimens that far away, or further, from the regression line. Instead there were 217 — over 5 times the number expected. There were 5 times the number of points expected ≥ 3 standard deviations away from the regression line for pure WSCT — 13 instead of the expected 2.6. Possible explanations could include data entry errors; different measurement technique for taking fork and total length in certain collections; and wide variation in the relationship between the two types of length measurements among the individual populations.

It is well beyond the scope of this project to sort out those possibilities; however the data are available to anyone who wishes to try. In the meantime, the regressions and their residual data can be used in the existing FWMIS data entry forms as a quality control technique to screen for likely data entry errors. Any records with fork length - total length combinations lying more than, say, 2.6 standard deviations away from the values predicted by these equations should be checked for errors in fork length, total length, or both. Ideally this would be done in the field while it is still possible to remeasure the fish. In the lab, the approach recommended by Chen and Jackson (2000) could be used to identify and deal with outliers.. At the least, these findings serve as a caution to field workers to take great care in making length measurements in a rigourously standardized fashion.

Despite these caveats, the regressions developed here provide reliable estimates with precision to within ± 4 mm in converting from one measurement to the other on individual fish, and within a few tenths of a millimetre for mean estimates. The low values for Cook's D provide confidence that the many outliers have not unduly influenced the regression parameters. The outliers become a major problem for the use of these regressions where highly accurate measurements are needed for specific populations which the residual analysis suggests could have different fork length - total length relationships than most. In such a case, separate regressions should be developed from careful measurements over the entire length range on fish within each such population.

There are three issues that this analysis hoped to resolve in the course of developing the regression equations. First, is there a difference between the OLS and the GM regression equations for converting between fork length and total length? Second, is there a difference between the equations derived from all CTTR and the equations derived from data on pure WSCT only? Finally, which is the best regression equation to use? The following answers apply only to data adequately represented by that analyzed here; that is, for converting between fork and total lengths of Alberta cutthroat trout within the range of lengths analyzed.

There is no difference between the OLS and GM equations for converting lengths for the two categories of cutthroat trout. The GM equations are indistinguishable from the OLS equations for converting between lengths of all CTTR. Similarly, the GM and OLS versions are effectively identical for converting lengths of pure WSCT. The reason for this is the high r^2 value (0.999+) for each equation: virtually all of the variance in each equation was accounted for by the regression. Under these conditions, the OLS and GM lines converge, becoming identical at $r^2 = 1$ (Jensen 1986).

The second question is more interesting. Within equation types (GM or OLS), there is a small but statistically significant difference between the conversion equations derived from the data for pure WSCT populations compared to those derived from the data for all CTTR populations. So, the GM conversion equations for pure WSCT are slightly different than the comparable equations for all CTTR as a whole. The OLS equations differ in the same manner. These differences, while statistically significant, are small, amounting to differences in length estimates of less than 0.5 mm, so are inconsequential for most purposes.

They may be of biological importance, however. The small differences may indicate a real morphological difference between hybrid cutthroats plus other subspecies, and genetically-pure westslope cutthroats. This should show up in a larger difference in the *a* intercepts if the same analysis is run after all known pure westslope cutthroats have been removed from the all-CTTR group. A consistent morphological difference of this type, even though very small, could serve as a means of identifying potentially pure westslope cutthroat populations that would be worth more detailed genetic sampling.

Trout length measurements are obtained by a variety of methods which are often not recorded in detail. For example, it is common for people to measure total length with the caudal fin compressed to maximize the measurement, but others measure total length to the end of the spread tail (Anderson and Neumann 1996:450). Trout measured on a stiff board do not show the same length as trout measured on a flexible plastic board, or with a tape held above the specimen lying on the ground. All of the pure WSCT populations were from the Oldman drainage and were sampled by relatively few biologists — mainly by Alberta Conservation Association, Fish and Wildlife staff and University of Lethbridge researchers. The all-CTTR populations were from throughout the eastern slopes of the Rocky Mountains by many different biologists, technicians, university researchers and private consultants. Because the many populations used in this study were sampled by a wide variety of people, not all of them perhaps trained to use standard methods, it is quite possible that some populations were measured by different methods. If the different types of measurements were differentially distributed between the pure WSCT and all CTTR groups, as seems likely, this could have caused the high rate of outliers, the differences in slopes between the regressions for those groups, or both.

In this study, the question of which conversion equation is best to use is fairly simple. For most practical purposes, it makes no real difference whether the GM or OLS equation is used: the results will differ negligibly. For example, using the OLS regression for all CTTR rather than the equivalent GM regression to convert a 200-mm fork length provides the following total length estimate.

210.2 mm, 95% CI 206.4 — 214.0 mm

This compares to the previous estimate using the GM equation for all CTTR (see Results):

210.1 mm, 95% CI 206.3 — 213.9 mm

This specimen would have had its total length recorded as 210 mm had it been directly measured in the field, exactly the same (to the same level of precision) as the estimate provided by either applicable GM regression. The comparable regressions for pure WSCT provide very similar estimates as well.

Still, it is best to use the most supportable equation for the nature of the data. Ricker (1973, 1984) provides guidance. The OLS regressions do not account for any variation associated with the independent variable, whereas there certainly is variation associated with both variables. The OLS regressions are less accurate for extrapolation beyond the length ranges of the data used to develop them, and are sensitive to nonrandom sampling such as that shown in the data of this study (small fish are seriously under-represented). OLS regressions are strictly valid only for the actual population sampled. The GM regression *does* account for variation in the independent variable, and is less sensitive to the under-representation of small fish as occurred here. It is easily calculated either directly, or from the equivalent OLS regression and the means of the dependent and independent variables. Published OLS regressions can easily be compared to the GM version in most cases.[5] Extrapolations to lengths beyond the length ranges of the data used to develop the GM regressions are robust.

For all of these reasons, the GM regressions are preferred for making length conversions of Alberta cutthroat trout. These should be applied to the length data from the appropriate population type: use the pure WSCT GM regressions for known pure WSCT, and the all-CTTR GM regressions for fish from populations which are known to be non-native subspecies or hybrids, or are of unknown genetic status.

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[5] GM regression can be easily derived from ordinary least squares regressions by making use of the 4th form of equation (14), b/r, and using equation (4) as above to obtain u. The required variable means and the r^2 statistic are commonly reported along with ordinary least squares regressions.

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Matthew Coombs of Alberta Environment and Sustainable Resource Development (AESRD), Blairmore, provided the cutthroat trout data from the FWMIS database, and Jennifer Earle of AESRD Cochrane contributed her key to genetic sampling locales and some new genetic data from Eric Taylor. Jennifer and Matthew also instigated the work through their perceptive comments on a draft of a different report. The study was funded as part of Fisheries and Oceans Canada Contract No. F2429-110038.

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Appendix Tables

Table A1. SYSTAT ordinary least squares regression analysis, analysis of variance, and residual analysis, all CTTR, total length on fork length.

```
DEP VAR:     TL      N:   14278  MULTIPLE R: 1.000  SQUARED MULTIPLE R: 0.999
ADJUSTED SQUARED MULTIPLE R: 0.999    STANDARD ERROR OF ESTIMATE:        1.952

VARIABLE      COEFFICIENT    STD ERROR     STD COEF TOLERANCE     T   P(2 TAIL)
CONSTANT           2.207        0.042        0.000      .      53.164    0.000
FL               1.040        0.000        1.000      1.000  .44E+04    0.000

                  ANALYSIS OF VARIANCE

SOURCE       SUM-OF-SQUARES   DF  MEAN-SQUARE     F-RATIO       P
REGRESSION     .724580E+08     1  .724580E+08 .190231E+08       0.000
RESIDUAL         54376.653 14276        3.809

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WARNING: CASE  3661 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3674 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.991)
WARNING: CASE  3685 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3685 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.695)
WARNING: CASE  3692 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.597)
WARNING: CASE  3706 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.706)
WARNING: CASE  3743 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3752 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.620)
WARNING: CASE  3756 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.675)
WARNING: CASE  3757 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3774 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3778 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3781 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3784 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3785 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3790 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3802 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3804 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3820 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3821 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.010)
WARNING: CASE  3824 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3827 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3829 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3833 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.721)
WARNING: CASE  3837 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3837 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.892)
WARNING: CASE  3838 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3841 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.667)
WARNING: CASE  3843 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
WARNING: CASE  3846 HAS LARGE LEVERAGE            (LEVERAGE =       0.001)
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WARNING: CASE  3846 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.475)
WARNING: CASE  3847 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3847 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.330)
WARNING: CASE  3848 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3849 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3856 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.617)
WARNING: CASE  3858 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.151)
WARNING: CASE  3862 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3865 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3887 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3888 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3889 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3894 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3897 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3899 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3901 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3901 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.188)
WARNING: CASE  3902 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3902 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.090)
WARNING: CASE  3909 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3919 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.172)
WARNING: CASE  3921 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3921 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.193)
WARNING: CASE  3962 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3962 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.229)
WARNING: CASE  3984 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  3987 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4011 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4011 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.942)
WARNING: CASE  4017 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.215)
WARNING: CASE  4026 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4027 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4037 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4041 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4041 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.414)
WARNING: CASE  4047 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4052 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4052 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.942)
WARNING: CASE  4060 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4061 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4070 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.778)
WARNING: CASE  4074 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4075 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4077 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.661)
WARNING: CASE  4083 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4138 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.638)
WARNING: CASE  4156 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4157 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.801)
WARNING: CASE  4158 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.149)
WARNING: CASE  4165 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4167 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4167 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.304)
WARNING: CASE  4171 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4190 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4195 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4199 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4200 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4205 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.338)
WARNING: CASE  4208 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4208 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.570)
WARNING: CASE  4218 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4219 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4222 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4222 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.009)
WARNING: CASE  4223 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.483)
WARNING: CASE  4230 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4237 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
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WARNING: CASE 4239 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4247 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4247 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.064)
WARNING: CASE 4255 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.745)
WARNING: CASE 4259 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4264 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.149)
WARNING: CASE 4267 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.047)
WARNING: CASE 4272 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4301 IS AN OUTLIER (STUDENTIZED RESIDUAL = -2.801)
WARNING: CASE 4302 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4312 IS AN OUTLIER (STUDENTIZED RESIDUAL = -4.688)
WARNING: CASE 4314 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4318 IS AN OUTLIER (STUDENTIZED RESIDUAL = -4.402)
WARNING: CASE 4319 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.027)
WARNING: CASE 4321 IS AN OUTLIER (STUDENTIZED RESIDUAL = -2.758)
WARNING: CASE 4322 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.665)
WARNING: CASE 4326 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4328 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.274)
WARNING: CASE 4330 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4330 IS AN OUTLIER (STUDENTIZED RESIDUAL = -5.509)
WARNING: CASE 4331 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.992)
WARNING: CASE 4334 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.417)
WARNING: CASE 4335 IS AN OUTLIER (STUDENTIZED RESIDUAL = -4.914)
WARNING: CASE 4337 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4337 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.168)
WARNING: CASE 4339 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4340 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4343 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4345 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4347 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4352 IS AN OUTLIER (STUDENTIZED RESIDUAL = 5.792)
WARNING: CASE 4353 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4356 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4360 IS AN OUTLIER (STUDENTIZED RESIDUAL = -2.781)
WARNING: CASE 4362 IS AN OUTLIER (STUDENTIZED RESIDUAL = 5.110)
WARNING: CASE 4367 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4377 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4379 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4382 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4383 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4387 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4388 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4389 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.315)
WARNING: CASE 4404 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4411 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4412 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4416 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4421 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4423 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4424 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4425 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4427 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4428 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4428 IS AN OUTLIER (STUDENTIZED RESIDUAL = -3.168)
WARNING: CASE 4431 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4433 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4434 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4444 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4446 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4447 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4448 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4449 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4453 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4457 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4460 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4462 HAS LARGE LEVERAGE (LEVERAGE = 0.001)
WARNING: CASE 4462 IS AN OUTLIER (STUDENTIZED RESIDUAL = -5.693)

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WARNING: CASE  4463 IS AN OUTLIER (STUDENTIZED RESIDUAL =     2.920)
WARNING: CASE  4464 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4465 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4466 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4475 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4478 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4487 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4499 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.667)
WARNING: CASE  4545 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.615)
WARNING: CASE  4554 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  4753 IS AN OUTLIER (STUDENTIZED RESIDUAL =     4.388)
WARNING: CASE  5113 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5133 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.657)
WARNING: CASE  5136 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5161 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5185 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5192 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5201 IS AN OUTLIER (STUDENTIZED RESIDUAL =     4.318)
WARNING: CASE  5205 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5209 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5209 IS AN OUTLIER (STUDENTIZED RESIDUAL =     3.314)
WARNING: CASE  5228 IS AN OUTLIER (STUDENTIZED RESIDUAL =     5.830)
WARNING: CASE  5271 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.784)
WARNING: CASE  5273 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -4.178)
WARNING: CASE  5277 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.638)
WARNING: CASE  5280 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.213)
WARNING: CASE  5281 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.763)
WARNING: CASE  5290 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.967)
WARNING: CASE  5293 IS AN OUTLIER (STUDENTIZED RESIDUAL =     2.667)
WARNING: CASE  5325 IS AN OUTLIER (STUDENTIZED RESIDUAL =     2.811)
WARNING: CASE  5331 IS AN OUTLIER (STUDENTIZED RESIDUAL =     4.230)
WARNING: CASE  5359 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.153)
WARNING: CASE  5437 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.761)
WARNING: CASE  5494 IS AN OUTLIER (STUDENTIZED RESIDUAL =     7.623)
WARNING: CASE  5553 IS AN OUTLIER (STUDENTIZED RESIDUAL =    10.104)
WARNING: CASE  5579 IS AN OUTLIER (STUDENTIZED RESIDUAL =     2.647)
WARNING: CASE  5683 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -4.363)
WARNING: CASE  5754 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.865)
WARNING: CASE  5755 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.759)
WARNING: CASE  5773 IS AN OUTLIER (STUDENTIZED RESIDUAL =     3.187)
WARNING: CASE  5783 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5798 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5813 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5858 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.678)
WARNING: CASE  5880 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.902)
WARNING: CASE  5890 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5907 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.279)
WARNING: CASE  5922 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.582)
WARNING: CASE  5924 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5937 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -5.511)
WARNING: CASE  5938 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.968)
WARNING: CASE  5941 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  5941 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -4.132)
WARNING: CASE  5999 IS AN OUTLIER (STUDENTIZED RESIDUAL =     6.778)
WARNING: CASE  6004 IS AN OUTLIER (STUDENTIZED RESIDUAL =     6.059)
WARNING: CASE  6046 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.093)
WARNING: CASE  6116 IS AN OUTLIER (STUDENTIZED RESIDUAL =    13.601)
WARNING: CASE  6146 IS AN OUTLIER (STUDENTIZED RESIDUAL =     5.196)
WARNING: CASE  6189 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -4.692)
WARNING: CASE  6357 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.131)
WARNING: CASE  6583 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -2.599)
WARNING: CASE  6707 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.437)
WARNING: CASE  6718 IS AN OUTLIER (STUDENTIZED RESIDUAL =    -3.232)
WARNING: CASE  6730 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  6748 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  6753 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE  6756 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
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WARNING: CASE  6756 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.064)
WARNING: CASE  6757 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6757 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.301)
WARNING: CASE  6758 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6758 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.454)
WARNING: CASE  6765 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.682)
WARNING: CASE  6767 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.959)
WARNING: CASE  6770 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.641)
WARNING: CASE  6786 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.361)
WARNING: CASE  6788 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.158)
WARNING: CASE  6795 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6797 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6799 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6804 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6805 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6806 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6809 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6812 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.757)
WARNING: CASE  6819 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6831 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6840 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  6848 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.230)
WARNING: CASE  6945 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.571)
WARNING: CASE  6991 IS AN OUTLIER (STUDENTIZED RESIDUAL =       2.979)
WARNING: CASE  7001 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.058)
WARNING: CASE  7028 IS AN OUTLIER (STUDENTIZED RESIDUAL =      11.056)
WARNING: CASE  7200 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.018)
WARNING: CASE  7454 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.119)
WARNING: CASE  7487 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.448)
WARNING: CASE  7491 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.656)
WARNING: CASE  7534 IS AN OUTLIER (STUDENTIZED RESIDUAL =       2.916)
WARNING: CASE  7540 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.988)
WARNING: CASE  7566 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.064)
WARNING: CASE  7568 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.675)
WARNING: CASE  7769 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.965)
WARNING: CASE  7776 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.865)
WARNING: CASE  7872 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.533)
WARNING: CASE  7910 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.748)
WARNING: CASE  7970 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.031)
WARNING: CASE  8271 IS AN OUTLIER (STUDENTIZED RESIDUAL =       2.974)
WARNING: CASE  8308 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  8312 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  8316 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.511)
WARNING: CASE  8318 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.867)
WARNING: CASE  8321 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.585)
WARNING: CASE  8330 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.902)
WARNING: CASE  8332 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.860)
WARNING: CASE  8337 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  8355 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  8366 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  8367 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -5.222)
WARNING: CASE  8382 IS AN OUTLIER (STUDENTIZED RESIDUAL =       3.923)
WARNING: CASE  8436 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.676)
WARNING: CASE  8496 IS AN OUTLIER (STUDENTIZED RESIDUAL =       5.520)
WARNING: CASE  8509 IS AN OUTLIER (STUDENTIZED RESIDUAL =       2.875)
WARNING: CASE  8517 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  8576 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.599)
WARNING: CASE  8700 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.253)
WARNING: CASE  8809 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.659)
WARNING: CASE  8818 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.924)
WARNING: CASE  8825 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.616)
WARNING: CASE  8831 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -3.026)
WARNING: CASE  8837 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -4.175)
WARNING: CASE  8841 IS AN OUTLIER (STUDENTIZED RESIDUAL =      -2.657)
WARNING: CASE  9113 IS AN OUTLIER (STUDENTIZED RESIDUAL =       2.769)
WARNING: CASE  9152 HAS LARGE LEVERAGE           (LEVERAGE =       0.001)
WARNING: CASE  9156 IS AN OUTLIER (STUDENTIZED RESIDUAL =       4.409)
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WARNING: CASE  9497 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.828)
WARNING: CASE  9535 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE  9544 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.304)
WARNING: CASE  9549 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.307)
WARNING: CASE  9674 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.687)
WARNING: CASE  9916 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.950)
WARNING: CASE 10051 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.747)
WARNING: CASE 10081 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.674)
WARNING: CASE 10224 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.872)
WARNING: CASE 10331 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.409)
WARNING: CASE 10507 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.128)
WARNING: CASE 10516 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.764)
WARNING: CASE 10520 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.905)
WARNING: CASE 10638 IS AN OUTLIER (STUDENTIZED RESIDUAL =      5.381)
WARNING: CASE 10672 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.697)
WARNING: CASE 10679 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -5.184)
WARNING: CASE 10704 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.134)
WARNING: CASE 10730 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -4.649)
WARNING: CASE 10937 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.733)
WARNING: CASE 11137 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.453)
WARNING: CASE 11200 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11206 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11207 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11209 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11212 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11214 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11215 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11218 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11220 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11224 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11227 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11228 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.611)
WARNING: CASE 11229 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.115)
WARNING: CASE 11246 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11247 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11252 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.675)
WARNING: CASE 11257 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11259 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11260 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11266 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11276 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11282 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11282 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.841)
WARNING: CASE 11305 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11312 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.021)
WARNING: CASE 11320 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11341 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.672)
WARNING: CASE 11381 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11384 IS AN OUTLIER (STUDENTIZED RESIDUAL =      5.551)
WARNING: CASE 11389 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11390 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.613)
WARNING: CASE 11402 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11403 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11403 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.904)
WARNING: CASE 11404 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.674)
WARNING: CASE 11411 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.778)
WARNING: CASE 11417 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11417 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.238)
WARNING: CASE 11435 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11438 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.744)
WARNING: CASE 11447 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11448 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11448 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.619)
WARNING: CASE 11460 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11463 HAS LARGE LEVERAGE           (LEVERAGE =      0.001)
WARNING: CASE 11465 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.131)
WARNING: CASE 11504 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.891)
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WARNING: CASE 11557 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11557 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.526)
WARNING: CASE 11578 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11578 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.808)
WARNING: CASE 11580 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11580 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -4.932)
WARNING: CASE 11581 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11583 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.596)
WARNING: CASE 11586 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11586 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.069)
WARNING: CASE 11590 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11591 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11597 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11598 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11601 HAS LARGE LEVERAGE          (LEVERAGE =      0.002)
WARNING: CASE 11601 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.492)
WARNING: CASE 11613 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11644 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11705 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.859)
WARNING: CASE 11707 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11708 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11709 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11710 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11713 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11715 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11716 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11717 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11719 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11720 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11721 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11723 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 11822 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.060)
WARNING: CASE 11870 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.835)
WARNING: CASE 11872 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.746)
WARNING: CASE 12113 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -4.198)
WARNING: CASE 12118 IS AN OUTLIER (STUDENTIZED RESIDUAL =      6.302)
WARNING: CASE 12204 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.657)
WARNING: CASE 12217 IS AN OUTLIER (STUDENTIZED RESIDUAL =      5.748)
WARNING: CASE 12348 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.911)
WARNING: CASE 12400 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.140)
WARNING: CASE 12427 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.243)
WARNING: CASE 12497 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.953)
WARNING: CASE 12610 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.172)
WARNING: CASE 12735 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.081)
WARNING: CASE 12776 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.768)
WARNING: CASE 12814 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.619)
WARNING: CASE 12910 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.720)
WARNING: CASE 13002 IS AN OUTLIER (STUDENTIZED RESIDUAL =      5.563)
WARNING: CASE 13079 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.811)
WARNING: CASE 13139 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.790)
WARNING: CASE 13148 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.806)
WARNING: CASE 13299 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.038)
WARNING: CASE 13315 IS AN OUTLIER (STUDENTIZED RESIDUAL =      6.386)
WARNING: CASE 13470 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.420)
WARNING: CASE 13535 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.286)
WARNING: CASE 13573 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.709)
WARNING: CASE 13599 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.307)
WARNING: CASE 13610 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.709)
WARNING: CASE 13754 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.474)
WARNING: CASE 13783 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.814)
WARNING: CASE 13891 IS AN OUTLIER (STUDENTIZED RESIDUAL =      5.952)
WARNING: CASE 13924 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.634)
WARNING: CASE 14196 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14204 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14205 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14207 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14207 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.373)
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WARNING: CASE 14222 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14222 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.023)
WARNING: CASE 14226 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14227 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14230 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14234 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14236 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.983)
WARNING: CASE 14237 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14239 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14240 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.540)
WARNING: CASE 14257 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.719)
WARNING: CASE 14265 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14266 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14267 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14267 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.664)
WARNING: CASE 14271 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14275 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14275 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -7.279)
WARNING: CASE 14276 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14277 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14277 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -4.604)
WARNING: CASE 14278 HAS LARGE LEVERAGE          (LEVERAGE =      0.001)
WARNING: CASE 14278 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -4.152)

DURBIN-WATSON D STATISTIC    1.580
FIRST ORDER AUTOCORRELATION  0.210
```

Table A2. SYSTAT ordinary least squares regression analysis, analysis of variance, and residual analysis, pure westslope cutthroat trout, total length on fork length.

```
DEP VAR:     TL     N:    926  MULTIPLE R: 1.000  SQUARED MULTIPLE R: 0.999
ADJUSTED SQUARED MULTIPLE R: 0.999    STANDARD ERROR OF ESTIMATE:       1.913
VARIABLE     COEFFICIENT    STD ERROR     STD COEF TOLERANCE    T   P(2 TAIL)
CONSTANT          1.759        0.159        0.000      .       11.077    0.000
FL              1.040        0.001        1.000     1.000  .11E+04    0.000

                  ANALYSIS OF VARIANCE
SOURCE      SUM-OF-SQUARES   DF  MEAN-SQUARE     F-RATIO       P
REGRESSION     4239900.776     1  4239900.776 1158255.577       0.000
RESIDUAL         3382.387   924       3.661

WARNING: CASE    73 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.883)
WARNING: CASE   166 IS AN OUTLIER (STUDENTIZED RESIDUAL =      3.332)
WARNING: CASE   175 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.989)
WARNING: CASE   203 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.769)
WARNING: CASE   216 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -4.420)
WARNING: CASE   229 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.853)
WARNING: CASE   232 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.811)
WARNING: CASE   253 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.323)
WARNING: CASE   256 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.725)
WARNING: CASE   279 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.755)
WARNING: CASE   284 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.917)
WARNING: CASE   286 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.300)
WARNING: CASE   298 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.726)
WARNING: CASE   333 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.113)
WARNING: CASE   335 HAS LARGE LEVERAGE          (LEVERAGE =      0.011)
WARNING: CASE   335 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.568)
WARNING: CASE   341 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.853)
WARNING: CASE   498 IS AN OUTLIER (STUDENTIZED RESIDUAL =      2.585)
WARNING: CASE   500 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.769)
WARNING: CASE   620 HAS LARGE LEVERAGE          (LEVERAGE =      0.010)
WARNING: CASE   624 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.790)
WARNING: CASE   724 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.011)
WARNING: CASE   838 IS AN OUTLIER (STUDENTIZED RESIDUAL =      4.790)
WARNING: CASE   901 HAS LARGE LEVERAGE          (LEVERAGE =      0.008)
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WARNING: CASE   902 HAS LARGE LEVERAGE          (LEVERAGE =      0.010)
WARNING: CASE   903 HAS LARGE LEVERAGE          (LEVERAGE =      0.008)
WARNING: CASE   904 HAS LARGE LEVERAGE          (LEVERAGE =      0.009)
WARNING: CASE   906 HAS LARGE LEVERAGE          (LEVERAGE =      0.012)
WARNING: CASE   907 HAS LARGE LEVERAGE          (LEVERAGE =      0.010)
WARNING: CASE   909 HAS LARGE LEVERAGE          (LEVERAGE =      0.012)
WARNING: CASE   909 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -3.249)
WARNING: CASE   914 HAS LARGE LEVERAGE          (LEVERAGE =      0.009)
WARNING: CASE   924 HAS LARGE LEVERAGE          (LEVERAGE =      0.016)
WARNING: CASE   924 IS AN OUTLIER (STUDENTIZED RESIDUAL =     -2.891)

DURBIN-WATSON D STATISTIC    1.225
FIRST ORDER AUTOCORRELATION  0.386
```

Table A3. Standard error of prediction (SEP) for *mean* estimated length by predicted length class, all CTTR. Use these for high-precision calculations of confidence limits on predicted *mean* lengths. Limits in parentheses are estimated from few fish in those length-classes. SEP for individual estimates as in Table 2.

CTTR all predicted TL, OLS		CTTR all predicted FL, OLS		CTTR all predicted TL, GM		CTTR all predicted FL, GM	
predicted length, mm	SEP mean, mm	predicted length, mm	SEP mean, mm	predicted length, mm	SEP mean, mm	predicted length, mm	SEP mean, mm
≤ 34.0	0.04	≤ 23.0	0.04	≤ 34.0	0.04	≤ 23.0	0.04
34.1 - 86.0	0.03	23.1 - 75.0	0.03	34.1 - 86.0	0.03	23.1 - 75.0	0.03
86.1 - 251.0	0.02	75.1 - 244.0	0.02	86.1 - 251.0	0.02	75.1 - 244.0	0.02
251.1 - 303.0	0.03	244.1 - 296.0	0.03	251.1 - 303.0	0.03	244.1 - 296.0	0.03
303.1 - 351.0	0.04	296.1 - 344.0	0.04	303.1 - 351.0	0.04	296.1 - 344.0	0.04
351.1 - 398.0	0.05	344.1 - 389.6	0.05	351.1 - 398.0	0.05	344.1 - 389.6	0.05
398.1 - 443.0	0.06	389.7 - (435.0)	0.06	398.1 - 443.0	0.06	389.7 - (435.0)	0.06
443.1 - (500)	0.07	(435.1 - 483)	0.07	443.1 - (500)	0.07	(435.1 - 483)	0.07
(> 500)	0.08	(> 483)	0.08	(> 500)	0.08	(> 483)	0.08

Table A4. Standard error of prediction for *mean* estimated length by predicted length class, pure WSCT. Use these for high-precision calculations of confidence limits on predicted *mean* lengths.

CTTR all predicted TL, OLS		CTTR all predicted FL, OLS		CTTR all predicted TL, GM		CTTR all predicted FL, GM	
predicted length, mm	**SEP mean, mm**	**predicted length, mm**	**SEP mean, mm**	**predicted length, mm**	**SEP mean, mm**	**predicted length, mm**	**SEP mean, mm**
≤ 305.0	0.1	≤ 298.4	0.1	≤ 305.0	0.1	≤ 298.4	0.1
> 305.0	0.2	> 298.4	0.2	> 305.0	0.2	> 298.4	0.2

Table A5. Standard error of prediction for *individual* estimated length by predicted length class, pure WSCT. Use these for high-precision calculations of confidence limits on predicted *individual* lengths. Limits in parentheses are estimated from few fish in those length-classes.

CTTR all predicted TL, OLS		CTTR all predicted FL, OLS		CTTR all predicted TL, GM		CTTR all predicted FL, GM	
single fish predicted length, mm	**SEP, mm**	**single fish predicted length, mm**	**SEP, mm**	**single fish predicted length, mm**	**SEP, mm**	**single fish predicted length, mm**	**SEP, mm**
≤ 214.0	1.91	≤ 297.0	1.84	≤ 214.0	1.91	≤ 297.0	1.84
214.1 - (400.0)	1.92	> 297.0	1.85	214.1 - (400.0)	1.92	> 297.0	1.85
(> 400.0)	1.93			(> 400.0)	1.93		